

Sections 5.4 & 5.5 Additional Practice (for homework!)

Here are all the identities I will give you on the test:

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \quad \cos u = \frac{1}{\sec u} \quad \tan u = \frac{1}{\cot u}$$
$$\csc u = \frac{1}{\sin u} \quad \sec u = \frac{1}{\cos u} \quad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

Co-function Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u$$
$$\cos\left(\frac{\pi}{2} - u\right) = \sin u$$
$$\tan\left(\frac{\pi}{2} - u\right) = \cot u$$
$$\cot\left(\frac{\pi}{2} - u\right) = \tan u$$
$$\sec\left(\frac{\pi}{2} - u\right) = \csc u$$
$$\csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \quad 1 + \tan^2 u = \sec^2 u \quad 1 + \cot^2 u = \csc^2 u$$

Odd & Even Identities

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$
$$\csc(-x) = -\csc x \quad \sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

Sum & Difference Formulas

$$\sin(\theta + \beta) = \sin\theta\cos\beta + \cos\theta\sin\beta$$

$$\sin(\theta - \beta) = \sin\theta\cos\beta - \cos\theta\sin\beta$$

$$\cos(\theta + \beta) = \cos\theta\cos\beta - \sin\theta\sin\beta$$

$$\cos(\theta - \beta) = \cos\theta\cos\beta + \sin\theta\sin\beta$$

$$\tan(\theta + \beta) = \frac{\tan\theta + \tan\beta}{1 - \tan\theta\tan\beta}$$

$$\tan(\theta - \beta) = \frac{\tan\theta - \tan\beta}{1 + \tan\theta\tan\beta}$$

Double-Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half-Angle Formulas

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1 - \cos\theta}{\sin\theta} = \frac{\sin\theta}{1 + \cos\theta}$$

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Using any of the above formulas, Find the exact value of the following (show your work):

1. $\sin\frac{19\pi}{12}$

2. $\cos 255^\circ$

3. $\tan\frac{13\pi}{12}$

4. $\tan 67.5^\circ$

5. $\cos\frac{5\pi}{12}$

6. $\tan 105^\circ$

7. There is no formula given for the cotangent of a sum or difference. How could you find the exact value of $\cot 15^\circ$?

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8. Find the values of the following, given $\tan p = \frac{1}{2}$, $\sec q = -\sqrt{5}$; p & q in QIII

$$\sin 2q =$$

$$\tan \frac{p}{2} =$$

$$\cos(p+q) =$$

$$\sin(p-q) =$$

9. Which expressions are equal to $\sin 15^\circ$? (There may be more than one correct choice)

A. $\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

B. $\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$

C. $\sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ$

D. $\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

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10. Simplify the following to one trig expression using any of the formulas:

$$\sin 3x \cos 2x + \cos 3x \sin 2x$$

$$\cos 37^\circ \cos 22^\circ - \sin 37^\circ \sin 22^\circ$$

$$\sin 10^\circ \cos 5^\circ + \cos 10^\circ \sin 5^\circ$$

$$\frac{\tan 5x - \tan 4x}{1 + \tan 5x \tan 4x}$$

$$\cos 5x \cos x - \sin 5x \sin x$$

$$\cos 15^\circ \cos 75^\circ - \sin 15^\circ \sin 75^\circ$$

$$\cos 4x \cos 3x + \sin 4x \sin 3x$$

$$\sin 2x \cos x - \sin x \cos 2x$$

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Find the solutions in the interval $[0, 2\pi)$ (think back to section 5.3)

11. $\sin 3x \cos 2x + \cos 3x \sin 2x = 1$

12. $\sin 2x = \sin x$

13. $\sin 2x - \cos x = 0$

14. $\cos 2x + 2 = -4 \cos x - 2 \cos^2 x$